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A METHOD OF DIGITALLY CONTROLLED FREQUENCY GENERATION.(U)

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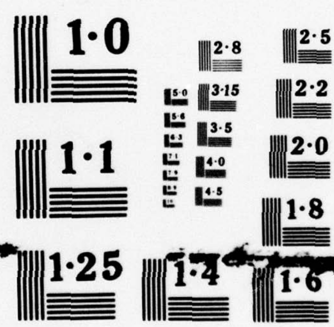
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A METHOD OF DIGITALLY CONTROLLED FREQUENCY GENERATION

by

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Ocean Technology Department

(12) 26p.

TN-183

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CONTENTS

INTRODUCTION

For years the advantages of being able to produce several frequencies from a single master source have been recognized. In many applications, the easiest method of producing different frequencies from a master clock has been to divide the clock frequency by stages to obtain a frequency which is a power-of-two division of the clock. This method has the intrinsic advantages of extreme simplicity, stability commensurate with the master clock, and precise repeatability. Its disadvantage is that only a very limited number of such frequencies may be produced.

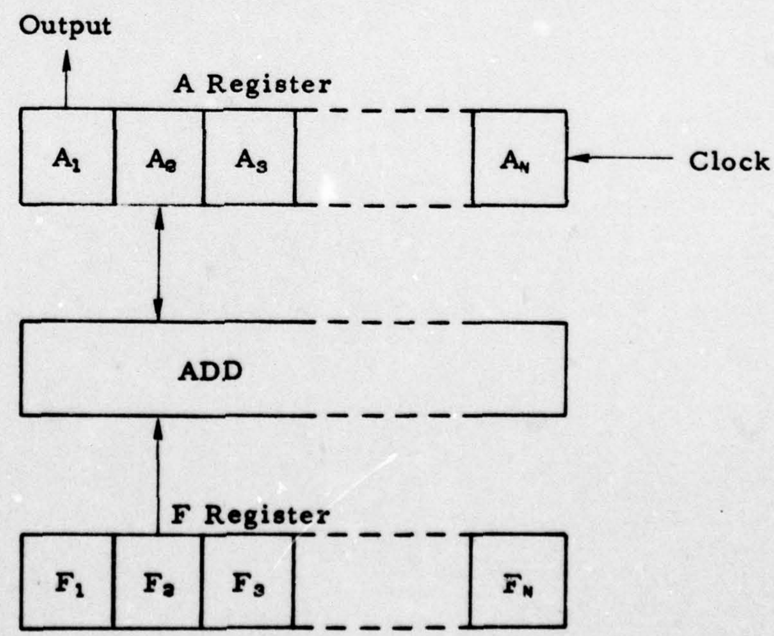
This paper describes a method by which a large number of precise, stable frequencies may be derived from a single frequency source.

SECTION I

DESCRIPTION

Imagine a register, A, of length N clocked at a frequency f_{clk} (Fig. 1). Imagine a second register, F, containing a number, M. The A register is to be initially set to zero. Then the contents of the F register are to be added to the contents of the A register at every clock pulse, and the result is to be stored in the A register. In other words, the A register is to be run as an accumulator. The rate of accumulation is determined by the number M in the F register.

Take as the output, the contents of the most significant Flip-Flop (FF), A_1, of the A register. Now the contents of the FF A_1 as a function of time may be regarded as a square wave whose frequency is determined by the number M carried in the F register. The larger the



The F register contains a number, M.

FIG. 1

number M, the faster the rate of accumulation, the sooner the A register will overflow, and the faster the A_1 FF will change state.

It will be clear that if the number M is a power of two, then the frequency of the square wave output will be the clock frequency divided by a power of two.

$M = 2^i$

Example:

Let

$$M = 1 \quad (M = 2^0)$$

The A register is able to contain the number $2^N - 1$. On the first clock pulse the number in the A register changes to 1. On the $2^N - 1$ n^{th} pulse the number changes to $2^N - 1$. On the 2^N th pulse, the A register overflows and the contents return to zero. Thus in 2^N pulses the A_1 FF has gone from zero to one and back to zero which is one cycle. The cycle is 2^N pulses long and the frequency is

$$f = \frac{f_{clk}}{2^N} \tag{1-1}$$

For M set equal to any power of two greater than zero, say 2^i, the A register is effectively shortened by i Flip-Flops, (Fig. 2), as all Flip-Flops to the right of the i^{th} FF never contain anything but zeros. The output frequency is now

$$f = \frac{f_{clk}\, 2^i}{2^N} = \frac{f_{clk}}{2^{N-i}} \tag{1-2}$$

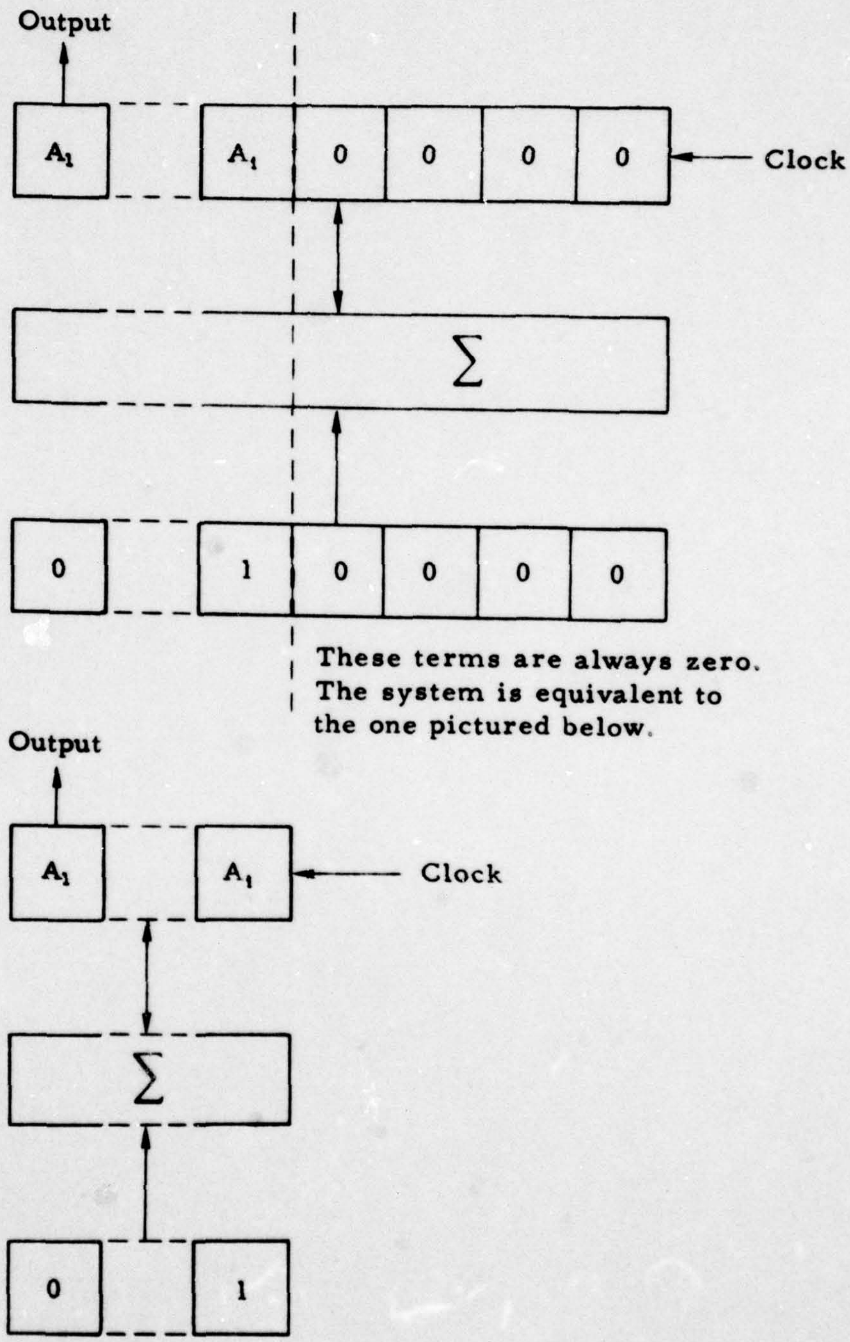

Output
A1
A1
0
0
0
0
Clock
∑
0
1
0
0
0
0
These terms are always zero.
The system is equivalent to
the one pictured below.
Output
A1
A1
Clock
∑
0
1

FIG. 2

ARBITRARY M

Now assume the F register to contain an arbitrary number, M. Then in general when the A register overflows, it does not return to zero, but to some positive value from which it begins a new accumulation.

Define

$I[x]$ = the integer part of x

$G[x]$ = the fractional part of x

Example:

$x = 3.4 \Rightarrow I[x] = 3; G[x] = 0.4$

If the A register begins at zero, the first overflow will occur in $I[2^N/M] + 1$ clock pulses and will leave a remainder, R, in the A register. This remainder, R, may be computed. Note that the number M has been added to the A register $I[2^N/M] + 1$ times. In other words an attempt has been made to place the number $M(I[2^N/M] + 1)$ in the A register. But the register will only hold the number 2^N. The remainder is the difference:

$$R = M(I[2^N/M] + 1) - 2^N \tag{1-3}$$

The A register will overflow in $I[2^N/M]$ pulses on the $k + 1^{st}$ cycle where

$$kR > M - R \tag{1-4}$$

and will continue to overflow in $I[2^N/M]$ cycles until the accumulated remainder goes to zero, i.e., in g cycles where

$$g(M - R) = kR \tag{1-5}$$

At this point, the A register contains the number zero and the entire process starts over again.

Actually, the process may not be quite so simple. The remainder may not go directly to zero, and the A register will go through some sequence of overflows in $I[2^N/M] + 1$ pulses and in $I[2^N/M]$ pulses until finally the remainder does go to zero (and eventually it must go to zero), whereupon the entire sequence repeats.

Each time the overflow occurs in $I[2^N/M] + 1$ pulses,

$$R = M(I[2^N/M] + 1) - 2^N \tag{1-6}$$

is added to the remainder. Each time the overflow occurs in $I[2^N/M]$ pulses,

$$M - R$$

is subtracted from the remainder. The entire sequence repeats when the remainder goes to zero, as that is the initial condition. If the total number of times the overflow occurred in $I[2^N/M] + 1$ pulses is given by k, and the total number of times the overflow occurred in $I[2^N/M]$ pulses is given by g, then it is clear that

$$g(M - R) = kR \tag{1-7}$$

as before.

Note that the fraction of the time that the overflow occurs in $I[2^N/M] + 1$ pulses is just

$$\frac{k}{g + k} = \frac{M - R}{M} = G[2^N/M] \tag{1-8}$$

The average frequency which is put out by FF A_1 is given by

$$f = \frac{Mf_{clk}}{2^N} \tag{1-9}$$

The average frequency is arrived at by the output switching between two frequencies, namely

$$f_{high} = \frac{f_{clk}}{I[2^N/M]} \tag{1-10}$$

and

$$f_{low} = \frac{f_{clk}}{I[2^N/M] + 1} \tag{1-11}$$

with the fraction of the time, t, spent at the latter frequency given by

$$t = G[2^N/M] \qquad \text{(Fig. 3)} \tag{1-12}$$

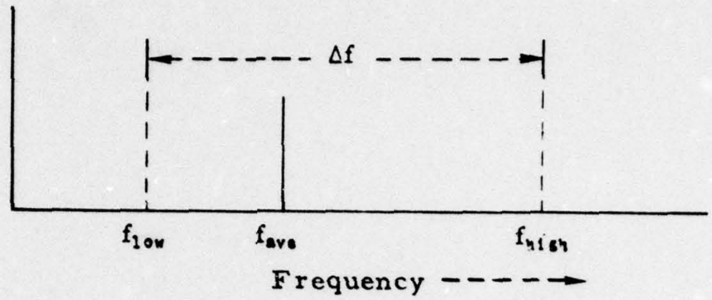
The separation between these two frequencies, Δf, is given by

$$\Delta f = \frac{f_{clk}}{I[2^N/M]^2 + I[2^N/M]} \tag{1-13}$$

and the fractional deviation between the two frequencies is given by

$$\frac{\Delta f}{f_{ave}} = \frac{2^N}{M(I[2^N/M]^2 + I[2^N/M])} \tag{1-14}$$

$$\frac{\Delta f}{f_{ave}} \cong \frac{M}{2^N} \quad \text{for } 2^N \gg M. \tag{1-15}$$



$$f_{low} = \frac{f_{clk}}{I[2^N/M] + 1}$$

$$f_{high} = \frac{f_{clk}}{I[2^N/M]}$$

$$f_{ave} = \frac{Mf_{clk}}{2^N}$$

$$\Delta f = \frac{f_{clk}}{I[2^N/M]^2 + I[2^N/M]}$$

FIG. 3

LIMITATIONS

The output waveform may be thought of as the zero-crossings of a sine wave of frequency

$$f = \frac{Mf_{clk}}{2^N} \quad (1\text{-}16)$$

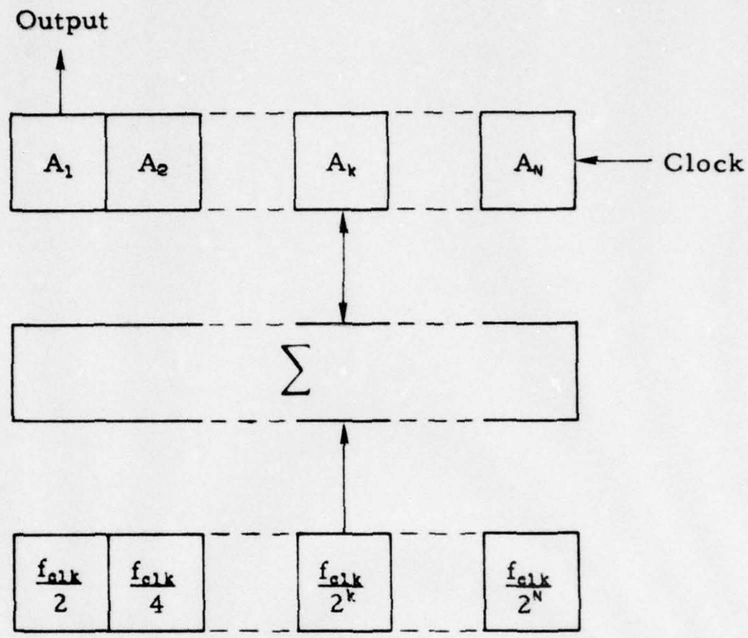
which is then sampled at the clock frequency. Indeed, this method <u>must</u> be considered as a sampled system. It should be clear that no frequency greater than $f_{clk}/2$ may be obtained, and no frequency lower than $f_{clk}/2^N$ may be obtained with a register of length N.

The precision to which any desired frequency may be produced is determined by the length of the A register. The frequencies which may be obtained are all multiples of $f_{clk}/2^N$, so that any frequency may be approached to within $f_{clk}/2^{N+1}$ Hz. A higher precision may be obtained by lengthening the A register.

In theory, any frequency below $f_{clk}/2$ may be approached to within any degree of accuracy, simply by making N sufficiently large. In practice, however, the maximum length of A, and therefore N_{max}, is determined by the circuitry used to perform the addition of the A and F registers. The carry produced by the sum of the least significant digits must be able to propagate through all the sums to the most significant digits during the time interval between adjacent clock pulses. This condition sets the ratio between f_{clk} and N, and effectively sets the upper limit on the range of frequencies which may be produced by this method.

One easy and simple method of determining the approximate frequency which will be obtained by any number in the F register is demonstrated in Fig. 4.

Each position in the F register corresponds to a frequency which is a power-of-two division of the clock. A "1" in any position will add the frequency corresponding to that position to the sum of the frequencies corresponding to the other "1"s in the F register.



The output frequency is given by

$f = \Sigma \frac{f_{clk}}{2^i}$ over all i's for which a "1" appears in the F register.

Example:

clock = 1 mHz

500k	250k	125k	62.5k	31k	15.5k	8k	4k
0	0	1	0	1	1	0	1

$f \cong 125 + 31 + 15.5 + 4 \text{ kHz} = 175.5 \text{ kHz}$

$f_{exact} = 175.781 \text{ kHz}$

FIG. 4

CONCLUSIONS

The great usefulness of this method of producing square-wave frequencies is its overwhelming simplicity. The frequencies which are produced are made with the repeatability and stability of the master clock, and may be identically phased simply by resetting the corresponding A registers together.

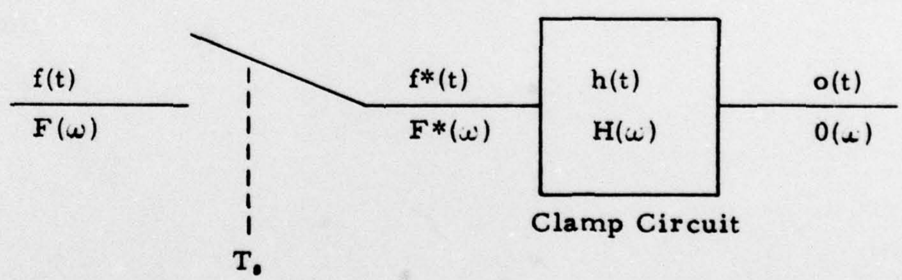
SECTION II

FREQUENCY SPECTRA

The description given thus far indicates that the output waveform is a square wave which contains frequency components that are not present in a "pure" square wave of the desired frequency. The equivalent circuit for the system is shown in Fig. 5. In order to analyze the output spectrum it would be advantageous at this time to derive an analytical expression for the spectrum of the output waveform, $0(\omega)$.

The input waveform is a "pure" square wave of the desired frequency, f(t). This function of time has a Fourier transform given by

$$\mathcal{F}[f(t)] = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{2-1}$$



h(t): 1 $\quad -T_0/2 < t < T_0/2$

0 everywhere else

FIG. 5

The function f(t) is sampled at times $T_s = 2\pi/\omega_s$ and the resulting waveform will be called f*(t), with Fourier transform $F^*(\omega)$. The function f*(t) is fed into a holding circuit whose output, o(t), with Fourier transform $0(\omega)$, is the function which the frequency generator actually produces.

Given an input function f(t), we have

$$f^*(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - nT_s) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s). \tag{2-2}$$

But

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{2-3}$$

can be written in a Fourier series as

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} e^{jm\omega_s t} \tag{2-4}$$

where

$$T_s = 2\pi/\omega_s \tag{2-4}$$

It follows that

$$f^*(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{2-5}$$

$$= \frac{1}{T_s} \sum_{m=-\infty}^{\infty} f(t)e^{jm\omega_s t} \tag{2-6}$$

and its Fourier transform is indicated by the formula

$$\mathcal{F}[f(t)e^{at}] = F(\omega - a) \tag{2-7}$$

so that

$$\mathcal{F}[f^*(t)] = F^*(\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} F(\omega - m\omega_s)^{1} \tag{2-8}$$

Thus it is shown that the spectrum of $f^*(t)$ is just the entire spectrum of $f(t)$ repeated at intervals of ω_s.

Given the following sequence to compute $F(\omega)$, the Fourier transform of $f(t)$:

if

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t} \tag{2-9}$$

where

$$\omega_0 = 2\pi/T$$

and

$$\alpha_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t} dt \tag{2-10}$$

then

$$F(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \alpha_n \delta(\omega - n\omega_0)^{2} \tag{2-11}$$

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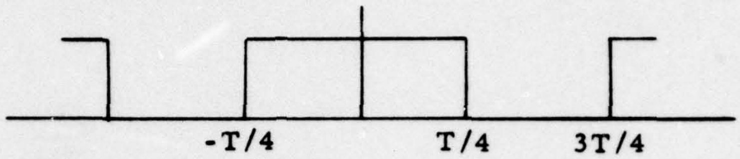
[1] Lindorff, David P., "Theory of Sampled-Data Control Systems," Wiley, N. Y., 1965, p. 30-31. The derivation given above follows Lindorff's text, but uses the notation of Papoulis cited below.

[2] Papoulis, Athanasios, "The Fourier Integral and Its Applications," McGraw-Hill, N. Y., p. 43.

The actual input waveform is



$$f(t) = 1 \quad -T/4 < t < T/4$$

$$= 0 \quad T/4 < t < 3T/4 \tag{2-12}$$

Assume

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t} \tag{2-13}$$

where

$$\omega_0 = 2\pi/T$$

then

$$\alpha_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt \tag{2-14}$$

$$\alpha_n = \frac{1}{T}\int_{-T/4}^{T/4} e^{-jn\omega_0 t}dt \tag{2-15}$$

$$= \frac{-1}{jTn\omega_0} e^{-jn\omega_0 t} \Big[_{-T/4}^{T/4} \tag{2-16}$$

$$= \frac{-4}{4jTn\omega_0} (e^{-jn\omega_0 T/4} - e^{jn\omega_0 T/4}) \tag{2-17}$$

$$= \frac{1}{2} \frac{\sin (n\omega_0 T/4)}{n\omega_0 T/4} \tag{2-18}$$

Substituting $T = 2\pi/\omega_0$

$$\alpha_n = \frac{\sin (n\pi/2)}{n\pi} \tag{2-19}$$

and combining this with Eq. 2-13 yields

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \sin (n\pi/2) e^{jn\omega_0 t} \tag{2-20}$$

and its Fourier transform is, from Eq. 2-11

$$F(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \sin (n\pi/2) \delta(\omega - n\omega_0). \tag{2-21}$$

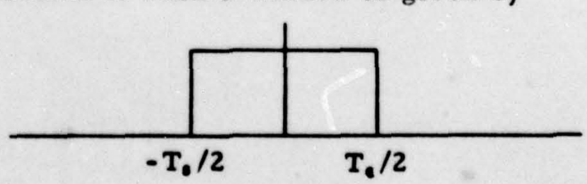
Thus from Eq. 2-8

$$F^*(\omega) = \frac{2\pi}{T_s} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \sin (n\pi/2) \delta(\omega - m\omega_s - n\omega_0) \tag{2-22}$$

Examining the clamp circuit, it is obvious that it has the property that for an impulse input, the output is a square pulse of a height equal to the impulse strength and of width $T_{hold} = T_s$. In other words, the output is the convolution of $\delta(t)$ and a square window of

$$T_{hold} = T_s = 2\pi/\omega_s. \tag{2-23}$$

The Fourier transform of such a window is given by



$$h(t) = 1 \quad -T_s/2 < t < T_s/2 \tag{2-24}$$

$$\qquad 0 \text{ everywhere else}$$

$$H(\omega) = \int_{-T_s/2}^{T_s/2} e^{-j\omega t} dt \tag{2-25}$$

$$= \frac{-1}{j\omega} e^{-j\omega t} \Big[_{-T_s/2}^{T_s/2} \tag{2-26}$$

$$= \frac{-1}{j\omega} (e^{-j\omega T_s/2} - e^{j\omega T_s/2}) \tag{2-27}$$

$$= \frac{2}{\omega} \frac{(e^{j\omega T_s/2} - e^{-j\omega T_s/2})}{2j} \tag{2-28}$$

$$= \frac{2}{\omega} \sin (\omega T_s/2) \tag{2-29}$$

$$H(\omega) = T_s \frac{\sin \omega\pi/\omega_s}{\omega\pi/\omega_s} \tag{2-30}$$

Now

$$0(\omega) = F^*(\omega) H(\omega) \tag{2-31}$$

$$\boxed{0(\omega) = 2\pi \frac{\sin \omega\pi/\omega_s}{\omega\pi/\omega_s} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \sin (n\pi/2) \delta(\omega - m\omega_s - n\omega_0)} \tag{2-32}$$

This result, while interesting as a complete expression of the output spectrum, is somewhat difficult to use in practice. Since the input waveform is a square wave, it contains spectral lines all the way out to infinity. It is clear that some of these lines will be folded about the line $\omega_s/2$ back down into the region below $\omega_s/2$. It would be useful to have an expression which bounds the fraction of the average power which

is folded down, and also one which bounds the most prominent spectral line which is folded back. Such an expression will now be derived. For any function of time which is expandable in a Fourier series, the average power in the waveform is given by:

$$\overline{f^2(t)} \equiv \text{average power} = \sum_{-\infty}^{\infty} |\alpha_n|^2 \quad [3] \tag{2-33}$$

$$\overline{f^2(t)} = \alpha_0^2 + 2 \sum_{1}^{\infty} |\alpha_n|^2 \qquad \text{by symmetry} \tag{2-34}$$

$$\alpha_n = \frac{1}{n\pi} \sin (n\pi/2) \tag{2-35}$$

$$\overline{f^2(t)} = \alpha_0^2 + \frac{2}{\pi^2} \sum_{\substack{1 \\ n \text{ odd}}}^{\infty} \frac{1}{n^2} \tag{2-36}$$

$$= \alpha_0^2 + \frac{2}{\pi^2} \left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\right) \tag{2-37}$$

$$= \alpha_0^2 + \frac{2}{\pi^2} \left(\frac{\pi^2}{8}\right) \quad [4] \tag{2-38}$$

$$= \alpha_0^2 + 1/4 \tag{2-39}$$

The average power in that portion of the waveform which is not folded is given by the finite summation:

$$\overline{f^2(t)}_s = \alpha_0^2 + \frac{2}{\pi^2} \sum_{\substack{1 \\ n \text{ odd}}}^{n_0} \frac{1}{n^2} \qquad n_0 : n_0\omega_0 < \omega_s/2 \tag{2-40}$$

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[3] Papoulis, op cit, p. 241

[4] Dwight, Herbert Bristol, "Tables of Integrals and Other Mathematical Data," Macmillan, N. Y., 1947, p. 11, Eq. 48.4.

So that now the fraction of the average power which goes into the unwanted spectrum which is folded back is given by

$$\overline{f^2(t)_u} = \overline{f^2(t)} - \overline{f^2(t)_s} \tag{2-41}$$

$$= 1/4 - \frac{2}{\pi^2} \sum_{1}^{n_0} \frac{1}{n^2} \quad \text{for n odd} \tag{2-42}$$

But this last summation is finite, and may be done for any given case in a reasonable length of time.

The fraction of the power which is folded back is bounded by the ratio of the unwanted power to the average power:

$$\frac{\overline{f^2(t)_u}}{\overline{f^2(t)}} = \frac{1/4 - \frac{2}{\pi^2} \sum_{1}^{n_0} \frac{1}{n^2}}{1/2} \qquad n_0 : n_0 \omega_0 < \omega_s/2 \tag{2-43}$$

This expression is an upper bound of the power which goes into the folded spectrum.

The most prominant line in the folded spectrum is just the lowest frequency line in the unwanted spectrum, namely that given by $n = n_0 + 2$. The ratio of the magnitude of this line to the magnitude of the fundamental of the square wave is just

$$\frac{1}{n_0 + 2} \tag{2-44}$$

SECTION III

EXAMPLES

The author recently had occasion to consider the following problem: produce 40 frequencies approximately 1 kHz apart between 30 kHz and 100 kHz. Available was a 4 mHz clock.

First, the length of the A register was determined: since the frequencies are to be about 1 kHz apart, and the frequency steps available from a register of length N is $f_{clk}/2^N$, set

$$\frac{f_{clk}}{2^N} \leq 1 \text{ kHz} \Rightarrow N \geq 12 \qquad (3\text{-}1)$$

This sets the length of the A register.

The F register must be able to produce numbers up to 100, as 100×1 kHz = 100 kHz is the highest frequency of interest. This sets the length of the F register at 7, with all higher order terms set to zero (Fig. 6).

Since the lower frequency limit is to be 30 kHz, set

$$\frac{Mf_{clk}}{2^N} \geq 30 \text{ kHz} \Rightarrow M = 31 \qquad (3\text{-}2)$$

This corresponds to a frequency of 30.273 kHz.

The next frequency is obtained by incrementing M by one, which produces a frequency of 31.250 kHz. This process is contined to the highest frequency, given by

$$M = 31 + 40 = 71 \qquad (3\text{-}3)$$

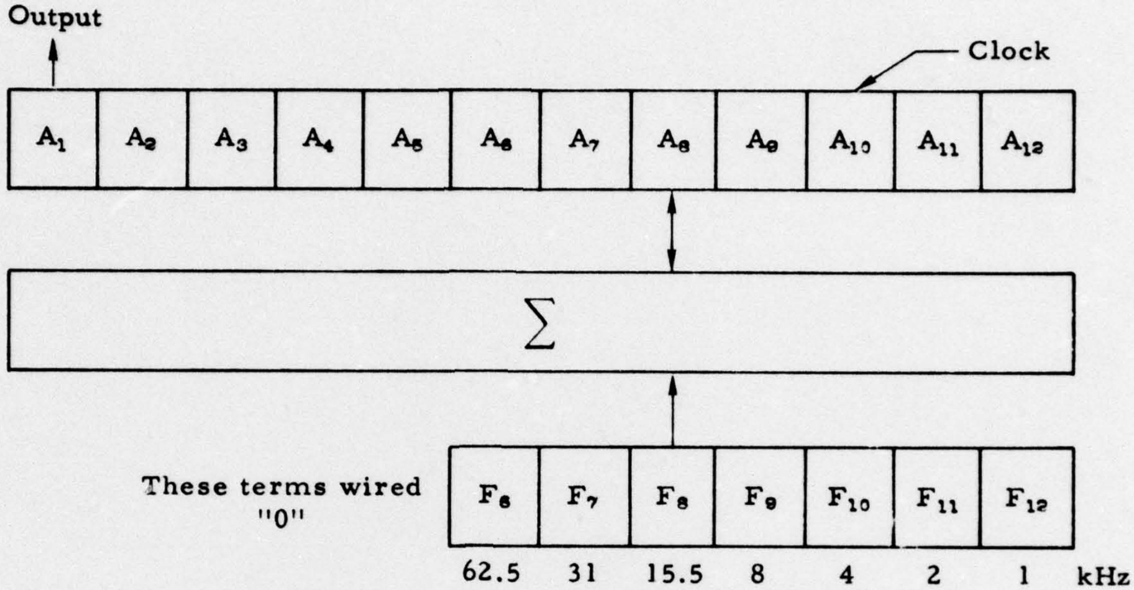

FIG. 6

which produces a frequency of 69.336 kHz. The separation between frequencies is 0.9765 kHz.

If this frequency separation were not sufficiently large, an alternate approach would be to make the A register a little longer and space the frequencies a little further apart. Set N = 14. Then the smallest frequency steps would be 244.14 Hz. The smallest M would be given by

$$M \geq \frac{30 \text{ kHz}}{0.24414 \text{ kHz}} = 122.8 \qquad (3\text{-}4)$$

Therefore set M = 123, which corresponds to a frequency of 30.029 kHz. The next frequency would be 31.250, an increase of 1.221 kHz, which is produced by M = 128 (an increase of 5). The highest frequency would be given by

$$M = 123 + 39(5) = 318 \qquad f = 77.636 \text{ kHz} \qquad (3\text{-}5)$$

Using Eq. 2-43, that percentage of the total power that is folded back may be calculated. This gives

$$n_0 : n_0 \omega_0 < \omega_s/2$$

$$\omega_0 = 77.636 \text{ kHz}$$

$$\omega_s/2 = 2{,}000 \text{ kHz}$$

$$n_0 < \frac{2{,}000}{77.636} = 25.761$$

Therefore, setting $n_0 = 25$

$$\sum_{1}^{n_0} \frac{1}{n^2}$$

is given by the following table:

n	$1/n^2$
1	1.00000
3	0.11111
5	0.04000
7	0.02041
9	0.01234
11	0.00826
13	0.00592
15	0.00444
17	0.00346
19	0.00277
21	0.00227
23	0.00189
25	0.00160
$\sum 1/n^2$ =	1.21447

Thus, Eq. 2-43 becomes

$$\frac{\overline{f^2(t)u}}{\overline{f^2(t)}} = \frac{1/4 - 2/\pi^2\ (1.21447)}{1/2}$$

$$= 0.5 - \frac{4.85788}{9.86959}$$

$$= 0.5 - 0.49221$$

$$= 0.00779$$

$$= -21 \text{ db}$$

And the most prominant line is down by 1/27 compared to the fundamental, or 0.0370 = -28.6 db.

Since the worst case is represented by the highest frequency made, the calculation above sets a maximum upper bound on the TOTAL folded power and the most prominant spectral line for all the frequencies in this example.

The author used Texas Instruments SN7483N Quad-Adders to perform the addition. These components have a propagation delay in the carry term of 12 nsec maximum per addition. The time between clock pulses (from trailing edge to leading edge) was set at 185 nsec. Thus the maximum number of additions which may be safely performed is 15. This determines the maximum length of the A register.

A number of frequency generators utilizing the above principles have been designed. Each frequency generator as actually constructed consists of 12 dual-in-line IC chips and occupies a volume of

$$V = 3\ 5/8 \times 2\ 1/8 \times 1/4 \text{ inches}^3 = 1.9 \text{ inches}^3.$$

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Dwight, Herbert Bristol. "Tables of Integrals and Other Mathematical Data," Macmillan, N. Y., 1947, p. 11 Eq. 48.4